On the nuclear states of K^- mesons

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Abstract

The $KNNN$ bound states recently discovered at KEK are studied. It is shown that the $\Lambda(1405)$ and $\Sigma(1385)$ resonant states coupled to the KN system may generate an attraction strong enough to form such bound states.

I. INTRODUCTION

The bound $KNNN$ states, recently discovered at KEK, open up a new field of research in the physics of nuclei with a strangeness content [1]. The state found in the K^-pnn system is bound by 195 MeV. It lives relatively long and decays into a hyperon and two nucleons. Nuclear states of kaons were expected, since early studies of kaonic atoms indicated that the nuclear potential for K^- mesons is attractive, see review [2]. On the other hand, the nuclear absorption related to the (K, π) conversion was found to be strong and such states would not be detectable due to their large ($\approx$ 100 MeV) widths. The existence of detectable narrow states was thus limited to some high angular momentum states. Another possibility was indicated in Ref. [3]. There it was shown that K^- mesons may be bound so strongly that most of the decay channels would be blocked. The mechanism of attraction was attributed to the $\Lambda(1405)$ state and narrow states of about 100 MeV binding were predicted to exist in large nuclei. That study was motivated by the measurement of an unusually strong repulsive level shift found in the 2P states of K^-He atoms [4]. Recently, the existence of narrow states was predicted, by Akaishi and Yamazaki, also in light nuclei such as ^{4}He, [5], and this prediction led to the experiment of Ref. [1]. Again, the mechanism of attraction was attributed to the strong isospin 0 attraction in the K^-p channel, which generates the $\Lambda(1405)$ as a K^-p bound state.

The actual experiment [1] shows that the attraction due to the $\Lambda(1405)$ apparently plays an important role but is not strong enough to generate the binding as observed. Both

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predictions [3] and [5] are based on a sizeable proton component in the nucleus and that is not the case in the K^-pnn system. In this letter it is shown that two resonant states $\Lambda(1405)$ and $\Sigma(1385)$ coupled to the KN system may generate the attraction required to produce the strong binding. This happens under two conditions:

- the $\Lambda(1405)$ and $\Sigma(1385)$ in a nuclear medium are located above the KN threshold.
- the binding of K^- is strong enough to block the main $\pi\Sigma$ and $\pi\Lambda$ decay modes.

Such conditions may be fulfilled in many nuclei, but the state generated with this interaction will be similar everywhere. The meson is bound in a restricted small area around the nuclear centre.

II. THE ORIGIN OF K^- ATTRACTION TO NUCLEI

To present the argument, let us assume the KN S-wave scattering amplitude to be described in terms of a simple resonance formula

$$f_o = \frac{\gamma_o^2}{E_{KN} - E_o + i\Gamma_o/2}, \tag{1}$$

where E_{KN} is the energy in the KN channel and E_o is a resonance energy. This amplitude is normalised to the scattering length at the threshold. For kaons in a nuclear medium it generates an optical potential

$$V_o(r) = \frac{4\pi}{2\mu_{KN}}\rho(r) f_o, \tag{2}$$

where μ_{KN} is the KN reduced mass and $\rho(r)$ is the nuclear density. The resonant situation produces an attractive potential when $E_{KN} - E_o < 0$ and a repulsive one otherwise. The question to settle is the value of the actual energy of the resonance in the nuclear medium. This is well understood in the $\Lambda(1405)$ case from the early days of K^- atom studies. The state is located below the KN threshold of 1432 MeV and leads to a repulsive K^-p scattering length. Contrary to that, an attraction is observed in kaonic atoms. It is generated by two effects: (1) the energy of the KN system is reduced by the nuclear binding and the KN recoil with respect to the nucleus [6]; (2) the position of the resonance is shifted upwards as a result of Pauli blocking, [7], [8], [9]. Both these effects create the $E_{KN} - E_o < 0$ situation which results in the attractive potential. While the kaonic atoms involve the low density nuclear surface region, for deep binding one needs the central densities. Calculations in Ref. [3] indicate that in nuclear matter the upward shift of $\Lambda(1405)$ creates a similar situation of an effective attraction. In the actual calculations Eq.(1) should be treated with more care as $\Lambda(1405)$ is to be described by a two channel K-matrix which generates it as a KN quasi-bound state [10], [11]. That results in a definite energy dependence of $\Gamma(E), E_o(E), \gamma(E)$, but the related physical effect remains the same.

The isospin structure of the kaonic states is given by

$$\mid K^-n >=\mid I=1>, \quad \mid K^-p >= \frac{1}{2}\mid I=1> + \frac{1}{2}\mid I=0> \tag{3}$$

and the $I=1$ channel involves another resonance strongly coupled to the KN channel in the subthreshold region. That is the $J=3/2$, $\Sigma(1385)$ which involves K^- interactions with

both protons and neutrons. In atomic states that resonance seems to play no role. However, in deeply bound states it becomes the dominant factor. The scattering amplitude

$$f_\Sigma = 2\mathbf{p}\mathbf{p}' \frac{\gamma^2_{\Sigma KN}}{E_{KN} - E_\Sigma + i\Gamma_\Sigma/2} \ , \tag{4}$$

where $\mathbf{p}, \mathbf{p}'$ are the relative momenta before and after the collisions, is normalised to the standard $l + 1/2$ partial wave which generates the factor 2. The resonant fraction gives the scattering volume. Inside the nuclear medium this term produces a dramatic effect on the kinetic energy of the meson. The disperison law in the nuclear medium becomes

$$E_K^2 - m_K^2 = p^2[1 + U_\Sigma(E_{KN})] + U_o \ , \tag{5}$$

where $U_o = 2m_K V_o$ is the S-wave contribution to the potential, $E_K = m_K - E_B$ and

$$U_\Sigma(E_{KN}) = \frac{4\pi\rho m_K}{\mu_{KN}} \frac{2\gamma^2_{\Sigma KN}(p^2)}{E_{KN} - E_\Sigma + i\Gamma_\Sigma(E_{KN})/2} \tag{6}$$

describes the P-wave interaction due to the $\Sigma(1385)$. This potential for energies E_{KN} close to, but less than, the resonance energy may be very large and attractive. Indeed, so large as to dominate the kinetic energy term and make negative the p^2 term in the dispersion formula. That happens for energy separations $E_{KN} - E_\Sigma$ of interest, at central nuclear densities and the coupling parameter which we discuss below.

The P-wave formula for $\Sigma(1385)$ coupling to the KN channel requires more subtle control over the energy dependence of the width and the formfactor in the resonance formula (6). The width is composed of three terms

$$\Gamma_\Sigma/2 = \Sigma_i \ p_i^3 \gamma_i^2(p_i^2) \ , \tag{7}$$

where the sum extends over the channels : $\pi\Sigma, \pi\Lambda, KN$. The resonant shape is observed in the $\pi\Lambda$ channel and the experimental width of 36 MeV gives the $\Sigma(1385)\Lambda\pi$ coupling constant, as the decays to the $\pi\Sigma$ channel constitute only 13% of the total, Ref. [14]. Below the KN threshold there is no contribution to the width coming from this channel. However, Eq. (7) generates a definite contribution $\mid p_{KN} \mid^3 \gamma^2_{KN}$ to the position of the resonance E_Σ. This contribution increases for energies below the resonance, making the real part of $E_{KN} - E_\Sigma + i\Gamma_\Sigma(E)/2$ stay small over a considerable range of energies. The coupling parameter $\gamma_{\Sigma\pi\Lambda}$ is known from the resonance width $\gamma^2_{\Sigma\pi\Lambda} = \Gamma_{\pi\Lambda}/(2p^3_{\pi\Lambda})$ and the corresponding coupling to the KN channel are related by SU(3) as $\gamma^2_{\Sigma KN}/\gamma^2_{\Sigma\pi\Lambda} = 2/3$. The experimental ratio of 0.51 ± 0.18 obtained from K^- capture in deuterium [15] is consistent with this SU(3) value. These numbers indicate $(1 + U_\Sigma)$ to be negative at central nuclear densities for $E_{KN} < E_\Sigma$ and $\mid E_{KN} - E_\Sigma \mid < 140$ MeV. As shown in the next section it is this effect that may generate the strong binding of K^- mesons. In the nuclear matter case it may generate a dangerous almost-collapse mechanism. The safe-guard against that situation is the resonance denominator itself. Large binding generates large $E_{KN} - E_\Sigma$ in Eq. (6) i.e. a weaker attraction and at the end a finite saturation is obtained. The actual saturation point depends on the position of the Σ resonance in the nuclear medium. As we show below it is likely to stay close to its free value.

The important point in question is the relative position of the KN threshold and the two resonances $\Lambda(1405)$ and $\Sigma(1385)$ situated in nuclear matter. Calculations [7], [8] produce an upward shift of the $\Lambda(1405)$. This shift can also be extracted from the experimental ratios of $\pi^+\Sigma^-/\ \pi^-\Sigma^+$ pairs produced by kaons stopped in nuclear emulsion. Such an analysis indicates a shift of about 10 MeV at the extreme nuclear surface [9]. Here we use an alternative, but fully consistent estimate. The coupling $G_{\Lambda KN}\Psi_N\Psi_\Lambda\psi_K$ may be described by a constant $G^2_{\Sigma KN}/4\pi \approx 1$ [13]. This generates a K-exchange potential between a proton and $\Lambda(1405)$ of an inverse range $\kappa = \sqrt{(M_\Lambda - M_K)^2 - m_K^2}$ and an average nuclear potential for the $\Lambda(1405)$ of strength $V_\Lambda = \rho_p G^2/\kappa^2 \approx 200MeV$ at the nuclear centres.

The shift of the $\Sigma(1385)$ follows from the gradient coupling $G_{\Sigma KN}\Psi_N\Psi^\mu_\Sigma\nabla_\mu\psi_K$. Such a coupling generates K-exchange between a nucleon and the Σ which flips the spin of the hyperon but also generates a diagonal term. The latter requires a correction at short nucleon Σ distances which is to be introduced in a way analogous to the subtraction of $\delta(r)$ in the π-exchange contribution to the nucleon-nucleon potential. The net potential, averaged over the nucleon densities, generates an attractive nuclear well for the $\Sigma(1385)$ with the strength $V_\Sigma = -\rho_1 G^2_{\Sigma KN}/3$. Here the density ρ_1 involves that component of the nuclear matter which couples to the hyperon in the isospin 1 state that is mostly neutrons. This potential is rather weak being about –30 MeV depth in nuclear matter. There is, unfortunately, no direct experimental check on this resonant shift.

III. THE K^-NNN SYSTEM

In the K^-pnn situation the $\Lambda(1405)$ may be formed on the proton. The average nuclear potential for the resonance vanishes within the K-meson exchange $\Lambda(1405)$-p interaction of the previous section. The $\Sigma(1385)$ is formed with much higher probability since, as seen from Eq.(3), it involves mostly neutrons. With the K-meson-exchange model one finds the nuclear well for the $\Sigma(1385)$ to be about -20 MeV deep.

With the K^- bound by some 200 MeV the relative separation of the KN threshold and the $\Lambda(1405)$ position amounts to 170 MeV and an extrapolation of the scattering amplitudes to this region is necessary. The K-matrix parametrization of A.Martin [11] ($K_{NN} = -1.65fm, R_{NN} = 0.18fm$) is used here, but it is supplemented by a separable model of Ref. [12] to obtain a smooth subthreshold extrapolation. This procedure generates the $I = 0$ scattering amplitude of $f_0 = 1.48$ fm, a fairly standard result in this energy region. As the energy of KN is so low, the pionic decay channels are closed and the scattering amplitudes are real. For the $I = 1$, K^-n amplitude the solution also from Ref. [11] is used ($K_{NN} = 1.07$ fm) which produces $f_1 = 0.34$ fm. These values generate a V_o, the potential well for K^-, of -110 MeV depth at the centre of the tritium nucleus. This is far too weak to produce the experimental binding. An additional and, in fact, predominant attraction comes from the $\Sigma(1385)$. It generates the potential described by a gradient term

$$U_G = \overleftarrow{\nabla}\ U_\Sigma(E_{KN})\ \overrightarrow{\nabla}\ , \tag{8}$$

where U_Σ is given by the resonant amplitude of Eq. (6).

Now we look for the variational solution for the kaonic energy level, $\epsilon = E_K^2 - m_K^2$,

$$\epsilon = Min \int d\mathbf{r}\Psi(r)[p^2 + \overleftarrow{\nabla} U_\Sigma(E_{KN}) \overrightarrow{\nabla} + U_o]\Psi(r). \tag{9}$$

In order to understand the nature of the solution and to guess the proper trial wave function we first solve a simpler problem. Consider a square well of radius R with the potential given by our local and gradient terms. In the internal region the wave equation

$$-[1 + U_\Sigma(E_{KN})]\psi_i''(r) + U_o\psi_i(r) = \epsilon\psi_i(r) \tag{10}$$

may, for some energies, be characterised by negative values of $[1 + U_\Sigma(E_{KN})]$. One has $\psi_i = \Psi/r = \sin(\kappa r)$ with $\kappa^2 = (-\epsilon + U_o)/(1 + U_\Sigma(E_{KN}))$. In the external region

$$-\psi_e''(r) = \epsilon\psi_e(r) \tag{11}$$

gives the asymptotic solution $\psi_e = C\exp(-\sqrt{-\epsilon}r)$, which has to be continuously matched to the internal solution that gives the eigenvalue condition $\kappa cot(\kappa R) = -\sqrt{-\epsilon}$. It leads to the solution

$$\epsilon = U_o + [1 + U_\Sigma(E_{KN})](\frac{\pi\xi}{R})^2 , \tag{12}$$

where ξ is a number in the range [0.5,1]. For negative and energy independent $1 + U_\Sigma$ the system collapses, since the minimal (and infinite) energy is obtained in the limit $R \to 0$. That is not the case with the realistic $U_\Sigma(E)$. Eq. (12) presents a nonlinear problem with respect to ϵ since $E_{KN} = M_N + m_K - E_B - E_{BN}$, where E_B, E_{BN} are the K^- and N binding energies. The minimum is found numerically and the solutions are always finite. With the parameters given in the text one obtains a trajectory for the binding energy $E_B(R) = -200$ MeV + R 27 MeV/fm, which is valid for radii R characteristic in tritium.

The minimal energy is obtained with $R \to 0$. This limit changes, when a formfactor is introduced in the coupling parameter $\gamma_{\Sigma KN}(p^2)$ of Eq. (6).

It is interesting to compare the K^-pnn system discussed above with the K^-ppn one. In the latter case, the central attraction is stronger due to the two protons that may generate the $\Lambda(1405)$. One obtains now $V_o(0) = -140$ MeV. On the other hand the $\Sigma(1385)$ gradient term is weaker by 25% due to the smaller $I = 1$ content, as given by Eq.(3). The K^- binding trajectory becomes $E_B(R) = -173$ MeV $+R$ 10 MeV /fm. The difference in the binding of these two systems may thus determine R that is the size of the region to which the K^- meson is confined.

Now the variational wave function is used in the form given by the square well model $\Psi = \psi/r$, and the free parameter of the variational procedure is R. The procedure itself is as follows:

1) For a given radius R the wave function Ψ is obtained.
2) The expected value of ϵ in Eq. (9) is calculated.
3) The kaon binding energy E_B is varied in the potential term $U_\Sigma(E_{KN})$ until selfconsistency with the variational binding is achieved.
4) The minimum of E_B with respect to R is found.

The radii R in the small region $R < 0.5$ fm generate essentially equivalent binding energies. A formfactor for $\Sigma(1385)$, $\gamma^2_{\Sigma KN}(p^2) = \gamma^2_{\Sigma KN}(0)/(1+(pr_o)^2)$ introduced into Eq. (6), locates the minimum at finite values of R. With the characteristic momenta of the confined

kaons $p \sim 1/R$ and $r_o \sim 0.5$ fm one obtains a very shallow minimum of $E_B(R)$ at $R = 0.4$ fm. The results are given in Table I. These include the K meson binding and a 7 MeV binding due to nucleons. The calculated energy is somewhat smaller than the experimental one. The difference may be due to many uncertainties in the physics involved. These are:

1) An uncertain subthreshold extrapolation of the resonant $I = 0$ and nonresonant $I = 1$ scattering amplitudes.

2) No experimental control of the $\Sigma(1385)$ energy in the nucleus.

3) An uncertain $\gamma_{\Sigma KN}$ coupling (an effect of this uncertainty is indicated in Table I).

4) Some restructuring of the three nucleon state.

Finally, more parameters in the trial wave function may reduce the variational result. We are not attempting any further discussion of these uncertainties. The aim of this letter is to show that the two KN subthreshold resonances are sufficient to bind the $KNNN$ system as strongly as observed. The width of such a state predicted in Ref. [3] is consistent with the experimental finding. It is based on the extrapolation (by the phase space) of the $KNN \rightarrow \Sigma(\Lambda)N$ decay rate known from the nuclear emulsion studies.

TABLES

TABLE I. The binding energies E_B, in MeV, of the K^-pnn and K^-ppn systems. First line is based on the $SU(3)$ value for the $KN\Sigma(1385)$ coupling. In the second line this coupling is enhanced by 20%.

$\gamma^2_{\Sigma KN}/\gamma^2_{\Sigma\pi\Lambda}$	K^-pnn	K^-ppn
2/3	169	155
1.2*2/3	192	175

IV. CONCLUSIONS

To summarise let us indicate again the main result, and suggest some topics for further research.

• The strange $\Lambda(1405)$ and $\Sigma(1385)$ baryonic states coupled to the KN system may generate the strongly bound K^- states, as is observed. Such K^- states, under normal nuclear density tend to be localised close to the nuclear centres. For much higher densities these are not necessarily the best solutions and the attraction due to the $\Sigma(1385)$ state may indicate a proximity to kaon condensation.

• The model developed here requires confirmation of some parameters, in particular the validity of the SU(3) symmetry in the $\Sigma(1385)$ couplings and the position of this resonance in nuclear media. An experiment indicating the $\Sigma(1385)$ decays in nuclei might elucidate this question.

• This model stresses the strengths of the K^-n interaction. The search for K^-nn, K^-nnn and other objects of neutron excess could be helpful.

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